Published by Institute of Physics Publishing for SISSA

Received: *July 13, 2006*
Accepted: *October 8, 2006*
Published: *October 24, 2006*

The α' stretched horizon in the heterotic string

Ghasem Exirifard

SISSA/ISAS
Via Beirut 2-4, I-34013 Trieste, Italy, and
INFN, Sezione di Trieste
Italy
Email: exir@sissa.it

Abstract: The linear α' corrections and the field redefinition ambiguities are studied for half-BPS singular backgrounds representing a wrapped fundamental string. It is showed that there exist schemes in which the inclusion of all the linear α' corrections converts these singular solutions to black holes with a regular horizon for which the modified Hawking-Bekenstein entropy is in agreement with the statistical entropy.

Keywords: Black Holes in String Theory, Superstrings and Heterotic Strings, Black Holes, Sigma Models.

Contents

1. Introduction **1**

2. The tree-level singular background **2**

3. The α' corrections **5**

4. Modification of the singularity **8**

5. Conclusions **14**

1. Introduction

The massless field of helicity two in the spectrum of string theory is identified as the gravitational field since its low energy effective action around flat space-time coincides with the Einstein-Hilbert action. This identification sets the subleading string corrections as the quantum corrections to gravity and allows one to ask if and how quantum corrections preserve or change the properties of classical backgrounds. In particular one may ask if the subleading string corrections induce a regular horizon on the singular classical geometries which have an entropy associated to them.

Amongst these singular classical geometries are the half BPS null singular ones which represent a wrapped fundamental string with general momentum and winding numbers [1]. These null singular geometries have a statistical entropy associated to them since string states with given momentum and winding numbers are degenerate [2]. It is conjectured that quantum effects convert these singular geometries to black holes with a regular horizon.

It is known that the the leading world-sheet corrections of the Heterotic string includes the square of the Riemann tensor. Ref [3], motivated by [4], observed that the inclusion of the square of the Riemann tensor and its supersymmetric partners in $D = 4$ [5−14] induces a local horizon with geometry $AdS_2 \times S^2$ on these backgrounds and for which the modified Hawking-Bekenstein entropy [15−17] is in agreement with the statistical entropy. This observation renewed interest in the subject [18−27]. Ref. [21, 28, 29] introduced the entropy formalism and concluded that the inclusion of the Gauss-Bonnet action as a part of the linear α' corrections in an arbitrary dimension induces a local horizon with geometry $AdS_2 \times S^{D-2}$ for which the modified Hawking-Bekenstein entropy is in agreement with the statistical entropy up to a numerical constant factor.

In this note we present a way to calculate all the linear α' corrections in an arbitrary dimension and we study how these corrections may change these null singular backgrounds to black holes. The note is organised in the following way;

In the second section we review the classical solutions representing a wrapped fundamental string on a cycle. We realise them as ten dimensional backgrounds composed of the metric, the NS two form and the dilaton first compacted on a torus of appropriate dimensionality to $D+1$ dimensional space-time and then through KK compactification on a circle to a D dimensional space-time.

In the third section we review how the α' corrections can be computed. We present the linear α' corrections in the Heterotic theory to backgrounds of metric, NS two form and dilaton obtained from scattering amplitude considerations [30, 31]. We study the field redefinition ambiguities. We require that the generalisation of the Einstein tensor is covariantly divergence free. This requirement fixes the curvature squared terms to the Gauss-Bonnet Lagrangian keeping some of the field redefinition ambiguity parameters untouched.

In the fourth section we discuss how the singularity could be modified by the inclusion of the α' corrections. We employ the compactification process of the first section to account for all the linear α' corrections in lower dimensions using the corrections in ten dimensions. We compute the local horizon configuration parameters for all field redefinitions compatible with ten dimensional diffeomorphism group. Note that the modified Hawking-Bekenstein entropy is the same for actions related to each other by field redefinition provided that the α' terms are studied as perturbations around a classical solution [32]. However since the local horizon is the exact solution of the truncated equations then the modified Hawking-Bekenstein entropy depends on the field redefinition ambiguity parameters. We show that there exist schemes in which the inclusion of all the linear α' corrections, however excluding the gravitational Chern-Simons term, in an arbitrary dimension gives rise to a local horizon with geometry $AdS_2 \times S^{D-2}$ for which the modified Hawking-Bekenstein entropy is compatible with the statistical entropy and outside which the higher order α' corrections could be perturbative. We also discuss on the existence of a smooth solution connecting the local horizon to asymptotic infinity.

In the fifth section the conclusions are presented.

2. The tree-level singular background

The low energy effective action of the critical heterotic string theory for the metric ($\boldsymbol{g}$), the NS two-form ($\boldsymbol{B}$) and the dilaton ($\boldsymbol{\phi}$) reads

$$\boldsymbol{S}^{(10)} = \frac{1}{32\,\pi} \int d^{10}\boldsymbol{x}\, \sqrt{-\boldsymbol{g}}\, e^{-2\boldsymbol{\phi}}\, \boldsymbol{L}^{(10)} \tag{2.1}$$

$$\boldsymbol{L}^{(10)} = (\boldsymbol{R}_{\text{Ricci}} + 4|\boldsymbol{\nabla}\boldsymbol{\phi}|^2 - \frac{1}{12}\boldsymbol{H}_{ijk}\boldsymbol{H}^{ijk})\,, \tag{2.2}$$

where

$$\boldsymbol{H}_{ijk} = 3\boldsymbol{B}_{[ij,k]}\,. \tag{2.3}$$

The bold symbols will be used to represent the fields in ten dimensions. Note that we are not using the modified field strength [33]

$$\boldsymbol{H}_{\text{modified}} = d\boldsymbol{B} - \frac{\alpha'}{4}[\frac{1}{30}\boldsymbol{\omega}_{3Y}(A) - \boldsymbol{\omega}_{3L}(\Omega)]\,, \tag{2.4}$$

where $\boldsymbol{\omega}_{3Y}(A)$ and $\boldsymbol{\omega}_{3L}(\Omega)$ stand for the Chern-Simons three-forms associated respectively to either the Spin(32)/Z_2 or $E8 \times E8$ connection and to the spin connection. In this work we are considering backgrounds of vanishing gauge connections where $\boldsymbol{\omega}_{3Y}(A) = 0$. The α' term in (2.4) represents a part of the linear α' corrections identified during the study of the anomaly cancellation. We shall study the effects of all the linear α' corrections however excluding the gravitation Chern-Simons term. For this purpose we use (2.3)

We are interested in the extrema of (2.1) whose fields configuration follows

$$\boldsymbol{ds^2} = \sum_{\mu,\nu=1}^{D} \boldsymbol{g}_{\mu\nu}(x)dx^\mu dx^\nu + 2\boldsymbol{g}_{y\mu}(x)\,dy\,dx^\mu + \boldsymbol{g}_{yy}(x)dy^2 + \sum_{m=D+1}^{10} dz_m^2\,, \tag{2.5}$$

$$\boldsymbol{B} = \boldsymbol{B}_{\mu\nu}(x)dx^\mu \wedge dx^\nu + \boldsymbol{B}_{y\mu}(x)dx^y \wedge dy\,, \tag{2.6}$$

$$\boldsymbol{\phi} = \boldsymbol{\phi}(x)\,, \tag{2.7}$$

$$y \sim y + 8\pi\,, \tag{2.8}$$

where y is compactified on a circle and z_i are compactified on T^{9-D}. These extrema are examples of trivial compactification on a torus of appropriate dimensionality from "10" dimensions to a "$D+1$" dimensional space-time and then KK compactification on a circle to a D dimensional space-time. If one represents non-trivial components of the ten dimensional fields by

$$\begin{aligned} \boldsymbol{g}_{yy}(x) &= T^2\,, & \boldsymbol{g}_{y\mu}(x) &= 2A^{(1)}T^2\,,\\ \boldsymbol{g}_{\mu\nu}(x) &= g_{\mu\nu} + 4T^2 A^{(1)}_\mu A^{(1)}_\nu\,, & \boldsymbol{B}_{y\mu}(x) &= 2A^{(2)}_\mu\,,\\ 2\boldsymbol{\phi}(x) &= 2\phi + \ln T - \ln V, & \boldsymbol{B}_{\mu\nu}(x) &= B_{\mu\nu} + 2(A^{(1)}_\mu A^{(2)}_\nu - A^{(1)}_\nu A^{(2)}_\mu)\,, \end{aligned} \tag{2.9}$$

where V is the volume of the compact directions. Then the induced action for the new fields - $g, A^{(1)}, A^{(2)}, B, T$ and ϕ- reads

$$\begin{aligned} S &= \int d^D x L \qquad\qquad (2.10)\\ &= \frac{1}{32\pi}\int d^D x\sqrt{-g}e^{-2\phi}(R_{\rm Ricci} + 4|\nabla\phi|^2 - \frac{|\nabla T|^2}{T^2} - \frac{|dB|^2}{12} - T^2|dA^{(1)}|^2 - \frac{|dA^{(2)}|^2}{T^2}), \end{aligned}$$

where $R_{\rm Ricci}$ is the Ricci scalar of $g_{\mu\nu}$, and an integration by parts is understood

$$\boldsymbol{L}^{(10)} - L = 2\sqrt{-g}\,\nabla^\mu(e^{-2\phi}\,\frac{\nabla_\mu T}{T})\,. \tag{2.11}$$

We refer to (2.10) as the induced action, and to x_μ and (z^μ,y) respectively as the large dimensions and as the compactified space. Due to the form of the induced action it is natural to interpret $A^{(1)}$ and $A^{(2)}$ as different U(1) gauge connections in the large dimensions.[1] A family of the extrema of the compactified action is given by

$$ds^2_{string} = -\,e^{4\phi(r)}\,dt^2 + dr^2 + r^2\,d\Omega^2_{D-2}\,, \tag{2.12}$$

[1]ref. ([34]) shows that the fields in large dimensions should be defined by (2.9) in order to not mix the U(1) symmetries.

$$e^{-4\phi(r)} = \frac{(r^{D-3}+2W)\,(r^{D-3}+2N)}{r^{2(D-3)}}\,, \qquad T(r) = \sqrt{\frac{r^{D-3}+2N}{r^{D-3}+2W}}\,, \tag{2.13}$$

$$A^{(1)}_{\tau}(r) = -\frac{N}{r^{D-3}+2N}\,, \qquad A^{(2)}_{\tau}(r) = -\frac{W}{r^{D-3}+2W}\,, \tag{2.14}$$

where N and W are two arbitrary numbers labelling the solution. We only consider the case where N and W are both positive. These backgrounds are constructed in [1] as singular limits of regular black-holes obtained by applying a solution generating transformation [35, 36] on a higher dimensional Kerr metric. Here we use the notation of [37]. ref. [1] proved that they break half of the ten dimensional supersymmetries leaving eight unbroken supersymmetry parameters. These backgrounds are null-singular, i.e. the horizon coincides with the singularity. They represent BPS states of an elementary string carrying n units of momentum and w units of winding charges along S^1 of the y coordinate where [2]

$$n = \frac{(D-3)\Omega_{D-2}}{4\pi}\,N\,, \tag{2.15}$$

$$w = \frac{(D-3)\Omega_{D-2}}{4\pi}\,W\,, \tag{2.16}$$

For general values of N and W a tachyon instability may exist around the singularity[3], reminiscent of the tachyon instability outside the horizon of Euclidean black holes presented in [38, 39]. We focus on the cases where this instability is not present.

An entropy may be associated to these backgrounds since in general there exists more than one state of the Heterotic string carrying w units of winding and n units of momentum along S^1 of the y coordinate. For large n and w the degeneracy of these states grows as $e^{4\pi\sqrt{nw}}$ [40]. Thus the entropy, defined by the logarithm of the degeneracy of the states, is given by:

$$S_{\text{statistical}} = 4\pi\sqrt{nw}\,, \tag{2.17}$$

when n and w are large. We refer to this entropy as the statistical entropy. A dilemma will arise as soon as the statistical entropy is associated to these tree-level backgrounds since they are singular and do not possess a regular event horizon to which the thermodynamical properties can be connected. This dilemma can be resolved in either of the following ways,

I. Statistical entropy should not be associated to these backgrounds.

II. Thermodynamical properties should be expressed in term of something else, in place of the event horizon, which null-singular geometries possess.

III. The subleading string corrections will induce an event horizon and the horizon cloaks the singularity.

[2] We have chosen a specific value for the radius of the compactification because the α' perturbative corrections to (2.12) do not depend on the radius of the compactification. The solution which represents KK-compactification on a circle with an arbitrary radius can be generated by rescaling y and using (2.9). This solution is written in [37].,sen2

[3] Localised winding tachyons may appear when a none-trivial cycle of the space-time shrinks to a point. The radius of the cycle of y coordinate at $r = 0$ becomes small for small $\frac{N}{W}$. Thus for general values of N and W some localised tachyon might appear around $r = 0$.

JHEP10(2006)070

Of the above possibilities, the first seems unnatural since the statistical entropy is associated to regular black holes [41 – 43] and these singular backgrounds are a limit of regular black holes. The fact that both the Euclidean path integral approach[4] [45] and the Noether current method [15, 16] express the entropy of a given black hole in term of its event horizon is not sufficient to conclude that entropy could not be associated to geometries without the event horizon. We would like to point out that Mathur and Lunin's description of the entropy [46] may resolve the dilemma in the second way. It is intersting that for the case of singular backgrounds representing D1-D5 branes, which have an entropy associated to them, both Mathur-Lunin description [47] and the subleading string corrections [27] can generate the entropy. In this note we study if the inclusion of subleading corrections can generate a horizon for backgrounds representing a fundamental string.[5]

3. The α' corrections

String theory provides two kind of perturbative corrections to a given background; the string loop corrections and the string world-sheet (α') corrections. The string coupling constant of (2.12), $g_s^2 = g_0^2 e^{2\phi}$, is

$$g_s^2 = g_0^2 \frac{r^{D-3}}{\sqrt{(r^{D-3} + 2W)(r^{D-3} + 2N)}} \leq g_0^2 \,, \tag{3.1}$$

where g_0 is an arbitrary parameter. We choose a sufficiently small value for g_0. Thus we ignore the string loop corrections. The α' corrections to the Lagrangian read

$$L \,=\, L^{(0)} \,+\, \alpha' \, L^{(1)} \,+\, \alpha'^2 \, L^{(2)} \,+\, \cdots \,, \tag{3.2}$$

where $L^{(0)}$ stands for the tree-level Lagrangian and the rest is its successive subleading corrections. This series may not make sense for (2.12) since each term of the α' series diverges at its singularity. However note that the α' corrections change the background itself

$$g \,\rightarrow\, g = g^{(0)} \,+\, \alpha' \, g^{(1)} \,+\, \alpha'^2 \, g^{(2)} \,+\, \cdots \,, \tag{3.3}$$

and the α'-corrected metric, possibly, can have a horizon outside which the α' expansion makes sense. The finiteness and smoothness of the α' corrections to the dilaton outside the horizon implies that the α' corrections to the string coupling constant, $g_s^2 = g_0^2 e^{2\phi}$, remain bounded outside the horizon. Hence if the induced horizon exists then the string loop corrections can be neglected consistently outside the horizon for sufficiently small values of g_0. In order to check the existence of the induced horizon we truncate the equations of motion at $O(\alpha'^2)$. Then we study if a exact solution of the truncated equations is a black hole with a regular horizon outside which the higher order α' corrections are perturbative. The perturbative α' corrections can be computed in the following ways

[4]Note that in string theory the presence of the tachyon-like winding modes of the tachyon wrapped around the Euclidean time which survive GSO projection [38, 39] adds to the known disturbing aspect [44] of the Euclidean approach.

[5]The subleading string corrections to the Schwarzschild black hole has been studied in [48, 49].

- From the scattering amplitudes of string on sphere as done in [30, 31, 50]. This method gives the Low Energy Effective action up to a perturbative field redefinition since field redefinitions do not alter the scattering amplitudes.
- Requiring exact conformal symmetry in the corresponding sigma model as done in [48, 51 − 55, 54]. In this method a regularisation and a renormalisation scheme should be chosen prior to computing the beta functions. Different schemes are related to each other by a perturbative field redefinition.
- Calculating the LEE action in the Heterotic closed string field theory [56]. This computation has not yet been accomplished. However it does not fix the perturbative field redefinition ambiguity since there remains the freedom to redefine the fields [57].

The first two methods give the same action up to a perturbative field redefinition ambiguity as the result of the consistency of the string theory around flat space-time [58, 59]. The outcome of the last method should be in agreement with those of the former ones. The linear α' corrections in Heterotic theory derived from string amplitude considerations read [30, 31]

$$\begin{aligned} S^{(1)}_{MT} &= \frac{1}{32\,\pi}\int d^{10}x\sqrt{-\det g}\;\frac{\alpha'}{8}\,e^{-2\boldsymbol{\phi}}\boldsymbol{L}^{(1)}_{MT}\,, \qquad (3.4)\\ \boldsymbol{L}^{(1)}_{MT} &= \boldsymbol{R}_{klmn}\boldsymbol{R}^{klmn}-\frac{1}{2}\boldsymbol{R}_{klmn}\boldsymbol{H}_p{}^{kl}\boldsymbol{H}^{pmn}+\\ &\quad -\frac{1}{8}\boldsymbol{H}_k{}^{mn}\boldsymbol{H}_{lmn}\boldsymbol{H}^{kpq}\boldsymbol{H}^l{}_{pq}+\frac{1}{24}\boldsymbol{H}_{klm}\boldsymbol{H}^k{}_{pq}\boldsymbol{H}_r{}^{lp}\boldsymbol{H}^{rmq}\,. \end{aligned}$$

This action beside the gravitational Chern-Simons modification of $d\boldsymbol{B}$ in $\boldsymbol{L}^(10)$ (2.1) includes all the linear α' corrections for backgrounds composed of the dilaton, the metric and the NS two form. A general field redefinition

$$\boldsymbol{g}_{ij} \to \boldsymbol{g}_{ij}+\alpha'\boldsymbol{T}_{ij}\,, \qquad (3.5)$$

$$\boldsymbol{B}_{ij} \to \boldsymbol{B}_{ij}+\alpha'\boldsymbol{S}_{ij}\,, \qquad (3.6)$$

$$\boldsymbol{\phi} \to \boldsymbol{\phi}-\alpha'\frac{\boldsymbol{X}}{2}\,, \qquad (3.7)$$

induces a change in $\boldsymbol{L}^{(1)}_{MT}$ of the form [60]

$$\begin{aligned} \Delta\boldsymbol{L} &= -\boldsymbol{T}^{ij}(\boldsymbol{R}_{ij}-\frac{1}{4}\boldsymbol{H}_{ikl}\boldsymbol{H}_j{}^{kl}+2\nabla_i\nabla_j\boldsymbol{\phi})+ \qquad (3.8)\\ &\quad +(\frac{1}{2}\boldsymbol{T}_i{}^i+\boldsymbol{X})(\boldsymbol{R}-\frac{1}{12}\boldsymbol{H}^2+4\nabla^2\boldsymbol{\phi}-4(\nabla\boldsymbol{\phi})^2)-\frac{1}{2}\nabla_k\boldsymbol{S}_{lm}\boldsymbol{H}^{klm}\,. \end{aligned}$$

where X, S_{ij} and T_{ij} are tensors with appropriate properties and are polynomials of $\boldsymbol{g}_{ij}$, $\boldsymbol{B}_{ij}$, $\boldsymbol{\phi}$ and their derivatives.[6] We consider only a class of the field redefinition ambiguities parameters given by

$$\boldsymbol{T}_{ij} = a\boldsymbol{R}_{ij}+\frac{b}{8}\boldsymbol{H}_{ikl}\boldsymbol{H}_j{}^{kl}+(e-12f)\,\boldsymbol{g}_{ij}\boldsymbol{R}+\,f\boldsymbol{g}_{ij}\boldsymbol{H}_{klm}\boldsymbol{H}^{klm}\,, \qquad (3.9)$$

[6]To compute $\Delta\boldsymbol{L}$ it is enough to remember that $\boldsymbol{g}^{ij}\delta\boldsymbol{R}_{ij}=(\boldsymbol{\nabla}^i\boldsymbol{\nabla}^j-\boldsymbol{g}^{ij}\Box)\delta\boldsymbol{g}_{ij}$. [61]

$$\boldsymbol{X} + \frac{1}{2}\boldsymbol{T}^i{}_i = (c-12f)\boldsymbol{R} + (\frac{d}{12}+3f)\boldsymbol{H}_{ijk}\boldsymbol{H}^{ijk}\,, \tag{3.10}$$

$$\boldsymbol{S}_{ij} = 0\,, \tag{3.11}$$

where a, b, c, d, e and f are real numbers. This class of field redefinition alters the linear α' corrected action by

$$\frac{1}{\alpha'}\Delta\boldsymbol{L} = -a\boldsymbol{R}_{ij}\boldsymbol{R}^{ij} + (c-e)\boldsymbol{R}^2 + (\frac{d}{12} - \frac{c}{12} + \frac{e}{4})\boldsymbol{R}\,\boldsymbol{H}^2 - \frac{d}{144}(\boldsymbol{H}^2)^2 \tag{3.12}$$
$$+(\frac{a}{4} - \frac{b}{8})\boldsymbol{H}^2_{ij}\boldsymbol{R}^{ij} + \frac{b}{32}\boldsymbol{H}^2_{ij}\boldsymbol{H}^{2ij} + O(\nabla\boldsymbol{\phi})\,,$$

where

$$\boldsymbol{H}^2_{ij} = \boldsymbol{H}_{ikl}\boldsymbol{H}_j{}^{kl}\,, \tag{3.13}$$

$$\boldsymbol{H}^2 = \boldsymbol{H}_{ijk}\boldsymbol{H}^{ijk}\,, \tag{3.14}$$

and the derivatives of the dilaton are not written to save space. In the forthcoming computations we do not need them. We require the generalisation of the Einstein tensor to be covariantly divergence free for a trivial dilaton. Adding this requirement to the linear α' corrections changes it to the first order Lovelock gravity [62] where $(a, c-e) = (\frac{1}{2}, \frac{1}{8})$.[7] Thus we set $(a, c) = (\frac{1}{2}, \frac{1}{8} + e)$ for which the linear α' corrected action reads

$$S = \frac{1}{32\pi}\int d^{10}x\,\sqrt{-\det\boldsymbol{g}}\,e^{-2\phi}\,\boldsymbol{L} \tag{3.15}$$

$$\boldsymbol{L} = \boldsymbol{R} - \frac{1}{12}\boldsymbol{H}^2 + 4|\boldsymbol{\nabla\phi}|^2 + \alpha'\boldsymbol{L}^{(1)} + \alpha' O(\boldsymbol{\nabla\phi}) + \alpha' O(\boldsymbol{\omega}_{3L}(\Omega)) + O(\alpha'^2) \tag{3.16}$$

$$\boldsymbol{L}^{(1)} = \frac{1}{8}\boldsymbol{L}_{GB} + \frac{1}{192}\boldsymbol{H}_{klm}\boldsymbol{H}^k{}_{pq}\boldsymbol{H}_r{}^{lp}\boldsymbol{H}^{rmq} - \frac{1}{16}\boldsymbol{R}_{klmn}\boldsymbol{H}_p{}^{kl}\boldsymbol{H}^{pmn} +$$
$$+(\frac{b}{32} - \frac{1}{64})\boldsymbol{H}^2_{ij}\boldsymbol{H}^{2ij} + (\frac{d}{12} - \frac{e}{6} - \frac{1}{96})\boldsymbol{R}\,\boldsymbol{H}^2 - \frac{d}{144}(\boldsymbol{H}^2)^2 + (\frac{1}{8} - \frac{b}{8})\boldsymbol{H}^2_{ij}\boldsymbol{R}^{ij}$$

where $\boldsymbol{L}_{GB} = \boldsymbol{R}_{ijkl}\boldsymbol{R}^{ijkl} - 4\boldsymbol{R}_{ij}\boldsymbol{R}^{ij} + \boldsymbol{R}^2$ is the Gauss-Bonnet term. In the work [63] and some follows works the α' corrections were required not to produce new extrema for the bi-linear part of the action describing deviation from flat Minkowski space. This criterion, the no-ghost criterion, is questionable since the new extrema are not perturbative in α'.[8] The criterion we used produces the same results and is independent of the perturbative behaviour of the α' series. However both of these criteria fail to identify a unique action.

[7] Lovelock gravity [62] is a generalisation of Einstein-Hilbert action where the generalisation of Einstein tensor G_{ij}: (1) is symmetric in its indices, (2) is a function of the metric and its first two derivatives, (3) is covariantly divergence free. The linear α' corrections can be chosen to satisfy all these conditions [63]. However the higher order α' corrections include also higher derivatives of the metric and can not be rewritten as higher order [64] Lovelock gravity [65].

[8] The spectrum of $R + \frac{\alpha'}{8}\boldsymbol{R}_{ijkl}\boldsymbol{R}^{ijkl}$ differs with the one of R when $\boldsymbol{R}_{ijkl}\boldsymbol{R}^{ijkl}$ is large. This implies that the ghosts are necessarily outside the perturbative regime.

JHEP10(2006)070

4. Modification of the singularity

We presume that there exists an exact α' background in the large dimensions which in the string frame reads

$$ds_{\text{exact}} = -f(r)dt^2 + dr^2 + g(r)d\Omega^2_{D-2} \tag{4.1}$$

$$\phi = \phi(r)\,, \qquad T = T(r)\,, \tag{4.2}$$

$$A_t^{(1)} = A_t^{(1)}(r)\,, \qquad A_t^{(2)} = A_t^{(2)}(r)\,, \tag{4.3}$$

the large r limits of which are (2.12), (2.13) and (2.14). The number of the modified supersymmetry charges[9] of this α' exact background should be the same as the number of SUSY charges of the tree-level background. It is conjectured [37] that this α' exact background has a regular event horizon with isometry group of $AdS_2 \times S^{D-2}$ whose fields in the vicinity of its horizon can be approximated by

$$ds^2 = v_1(-\rho^2 d\tau^2 + \frac{d\rho^2}{\rho^2}) + v_2 d\,\Omega^2_{D-2}\,, \tag{4.4}$$

$$e^{-2\phi(\rho)} = s\,, \tag{4.5}$$

$$T(\rho) = T\,, \tag{4.6}$$

$$F^{(1)}_{t\rho} = e_1\,, \tag{4.7}$$

$$F^{(2)}_{t\rho} = e_2\,, \tag{4.8}$$

where v_1, v_2, s, T, e_1 and e_2 are constant real (s, T are positive) numbers to be fixed by the equations of motion and the behaviour of the fields at infinity. A concrete proof or refutal of this conjecture requires knowing all the α' corrections. Neither the string scattering amplitudes nor the sigma model techniques nor CSFT are practically useful to compute the infinite terms of the α'-expansion series. There exists no other known method capable of producing the full α'-corrected action.[10] Currently the conjecture is supported by

I. Inclusion of only the Gauss-Bonnet action in the induced action allows for the existence of a local horizon geometry whose modified thermodynamical entropy [15 – 17] is in agreement with the statistical entropy up to a numerical constant [37].

II. Inclusion of $R_{ijkl}R^{ijkl}$ and the terms needed by SUSY [5 – 14] in the four dimensional induced action allows for a local horizon whose modified thermodynamical entropy is in agreement with the statistical entropy [3]. In higher dimensions it is not known which terms should be added to $R_{ijkl}R^{ijkl}$ to maintain SUSY.

The conjecture may be contradicted by:

I. The fundamental string is a special case of the null sigma models [69, 70]. It means that there exists a scheme in which the background fields retain their forms at the

[9] In LEEA the supersymmetry is realised as the symmetry of the action therefore, at least, the on shell SUSY constraints needs modification upon the inclusion of the subleading corrections.

[10] There have been attempts to guess a compact form for the α' expansion series of the metric [67, 68].

supergravity approximation. Thus within this scheme the fundamental string remains as a null singular background even after the inclusion of all the α' corrections. Does this contradict the appearance of a horizon due to the inclusion of the α' corrections?

II. The value of Wald's entropy is invariant under field redefinition provided that the α' terms are studied as perturbations around a classical background [32]. Here since Wald's formula is applied on the local horizon which is the exact solution of the truncated equations of motion then Wald's entropy depends on the field redefinition ambiguity parameters. Therefore which values should be chosen for the field redefinition parameters to calculate Wald entropy?

III. The Gauss-Bonnet action or the supersymmetric version of curvature squared terms are not all the linear α' corrections. This fact was also pointed out in [24]. Does the inclusion of all the linear α' corrections allow for the existence of the horizon?

IV. Is there a smooth interpolating solution from the horizon toward the asymptotic infinity?

V. Could the higher order α' corrections be consistently neglected?

The existence of a scheme with no α' corrections does not exclude existence of a scheme which converts a wrapped fundamental string into a regular black hole. It is not known which scheme would be preferred by the underlying conformal field theory.[11] We will show that inclusion of all linear α' corrections, however excluding the gravitational Chern-Simon term, produces the local horizon. We illustrate that in general the modified Hawking-Bekenstein entropy associated to the local horizon is not the same for actions related to each other by field redefinitions. Amongst these actions, the choices for which the modified Hawking-Bekenstein entropy is in agreement with the statistical entropy would be preferred. We show that in some schemes the higher order corrections can be ignored outside the α' stretched horizon.

We obtain the linear α' corrections to the induced action by applying the compactification process to the linear α' corrected action in ten dimensions (3.15). We consider the linear α' corrected action in (3.15) for all values of the field redefinition parameters, (b, d, e, f). ref. [71] shows that the pull back of (3.8) to the four dimensional space time is a functional of the gauge field strengths of A_1 and A_2. Thus we can use the entropy formalism [21, 29] to express the local horizon parameters in terms of the electric charges. The entropy formalism utilises the entropy function defined by

$$f(\vec{v}, T, \vec{e}) = \frac{1}{32\,\pi} \int d\theta\, d\phi\, \sqrt{-\det g}\, s\, L(\vec{v}, T, \vec{e}) \tag{4.9}$$

[11] ref. [83, 84] prove that for a regular horizon of geometry $AdS_2 \times S^D$ in Heterotic theories all the corrections to the thermodynamical entropy could be reproduced by the inclusion of the Gauss-Bonnet action to the induced action in the supergravity approximation. This renormalisation theorem does not state that there is an induced horizon with geometry $AdS_2 \times S^D$.

JHEP10(2006)070

where $L(\vec{v}, T, \vec{e})$ is the induced Lagrangian evaluated on the horizon configuration,

$$S = \frac{1}{32\,\pi} \int d^4x\,\sqrt{-\det g}\, e^{-2\phi}\, L(\vec{v},\, T,\, \vec{e}). \tag{4.10}$$

Then the equations of motions are equivalent to

$$\frac{\partial f}{\partial v_i} = 0\,, \tag{4.11}$$
$$\frac{\partial f}{\partial s} = 0\,, \tag{4.12}$$
$$\frac{\partial f}{\partial T} = 0\,, \tag{4.13}$$
$$\frac{\partial f}{\partial e_1} = \frac{N}{2}\,, \tag{4.14}$$
$$\frac{\partial f}{\partial e_2} = \frac{W}{2}\,, \tag{4.15}$$

where we have used the notation of Appendix A of [29] for the normalisation of the charges. To evaluate the induced action near the horizon we employ (2.9) to reconstruct the horizon configuration in ten dimensions from (4.4)-(4.8)[12]

$$\begin{aligned} d\boldsymbol{s}^2 &= ds^2 + T^2(dy + 2\,e_1\,r\,d\tau)^2 + \sum dz_i^2, \\ e^{-2\boldsymbol{\phi}} &= \frac{s}{T}\,, \\ \boldsymbol{B} &= -2\,e_2\,r\,d\tau \wedge\, dy\,. \end{aligned} \tag{4.16}$$

where the gauges are fixed by

$$A_1 = (e_1\,r, 0, 0, 0)\,, \tag{4.17}$$
$$A_2 = (e_2\,r, 0, 0, 0)\,. \tag{4.18}$$

Note that the class of field redefinitions considered in (3.12) includes any field redefinition which produces non-zero terms in the action near the horizon (4.16) and whose metric and NS two-form equations of motion are second order differential equations. For the sake of simplicity from this time on we set $D = 4$ and we study the four dimensional background representing the fundamental string,

$$D = 4\,. \tag{4.19}$$

Using the ten dimensional background near the horizon (4.16) one finds that

$$L_0 = \boldsymbol{R} - \frac{1}{12}\boldsymbol{H^2} = -\frac{2}{v_1} + \frac{2}{v_1} + \frac{2\,{e_1}^2\,T^2}{{v_1}^2} + \frac{2\,{e_1}^2}{{v_1}^2\,T^2} \tag{4.20}$$

$$L_1 = \frac{1}{8}\boldsymbol{L}_{GB} = -\frac{1}{v_1\,v_2} + \frac{T^2\,{e_1}^2}{{v_1}^2\,v_2} \tag{4.21}$$

[12] The compactification of the Gauss-Bonnet action has been done in [72].

$$L_2 \;=\; \frac{1}{192}\boldsymbol{H}_{klm}\boldsymbol{H}^{k}{}_{pq}\boldsymbol{H}_{r}{}^{lp}\boldsymbol{H}^{rmq} \;=\; \frac{{e_2}^4}{2\,{v_1}^4\,T^4} \tag{4.22}$$

$$L_3 \;=\; -\frac{1}{16}\boldsymbol{R}_{klmn}\boldsymbol{H}_{p}{}^{kl}\boldsymbol{H}^{pmn} \;=\; \frac{{e_1}^2\,{e_2}^2}{{v_1}^4} \;-\; \frac{{e_2}^2}{{v_1}^3\,T^2} \tag{4.23}$$

$$L_4 \;=\; (\frac{b}{32}-\frac{1}{64})\,\boldsymbol{H}^2_{ij}\boldsymbol{H}^{2ij} \;=\; 6\,(b-\frac{1}{2})\frac{{e_2}^4}{{v_1}^4\,T^4} \tag{4.24}$$

$$L_5 \;=\; (\frac{1}{8}-\frac{b}{8})\boldsymbol{H}^2_{ij}\boldsymbol{R}^{ij} \;=\; 2\,(b-1)\,(\frac{{e_1}^2\,{e_2}^2}{{v_1}^4} \;-\; \frac{{e_2}^2}{{v_1}^3\,T^2}) \tag{4.25}$$

$$L_6 \;=\; (\frac{d}{12}-\frac{e}{6}-\frac{1}{96})\boldsymbol{R}\,\boldsymbol{H}^2 \;=\; h\,{e_2}^2\,(\frac{1}{v_1^3\,T^2} \;-\; \frac{{e_1}^2}{{v_1}^4} - \frac{1}{{v_1}^2\,v_2\,T^2}) \tag{4.26}$$

$$L_7 \;=\; \frac{d}{144}\,(\boldsymbol{H}^2)^2 \;=\; 4\,d\,\frac{{e_2}^4}{v_1^4\,T^4} \tag{4.27}$$

where we used h defined by $h \;=\; 4d - 8e - \frac{1}{2}$ to represent L_6 in a more convenient way. Inserting the above expressions in ten dimensional action we get

$$S \;=\; \boldsymbol{S} \;=\; \frac{1}{32\,\pi}\int dt\,dr\,d\phi\,d\cos\theta\;s\;v_1\;v_2\,(L_0\;+\;\alpha'\sum_{i=1}^{7}L_i) + O(\alpha'^2)\,, \tag{4.28}$$

where the integration over the compact space is understood. Then the entropy function follows

$$f(\vec{v},\vec{e},s,T) \;=\; \frac{1}{8}\,s\,v_1\;v_2(L_0\;+\;\alpha'\sum_{i=1}^{7}L_i) \tag{4.29}$$

where we have truncated the α' series. Using (4.29) in (4.11)-(4.15) gives the equations of motion. The solution of the equations of motion identifies the horizon parameters. The identification of the near horizon geometry of half BPS backgrounds is an example of the supersymmetric attractor mechanism [73, 74], where the explicit equations of motion are solved rather than the supersymmetric constraints. Solving the equations of motion was first carried out by Ashoke Sen in [37] where only the Gauss-Bonnet Lagrangian was included in the induced action. The Gauss-Bonnet Lagrangian in the four dimensions reads

$$\frac{1}{8}(R_{ijkl}R^{ijkl} - 4R_{ij}R^{ij} + R^2) \;=\; -\frac{1}{v_1\;v_2} \tag{4.30}$$

which coincided with the first term in L_1. We see that in total five terms in the the summation of $L_1 + \cdots + L_7$ are not reproduced by the inclusion of the four-dimensional Gauss-Bonnet Lagrangian.

A linear combination of the equations of motion of T and of v_1 factorises

$$\frac{\partial f}{\partial s} = 0\;\rightarrow f\;=\;0\,, \tag{4.31}$$

$$(\frac{1}{T}\frac{\partial f}{\partial T} - 4{e_1}^2\frac{\partial f}{\partial v_1})\,|_{f=0} \;=\; (T^2{e_1}^2 - \frac{{v_1}^2}{4})(\ldots)\,. \tag{4.32}$$

eq. (4.32) implies that some of the solutions may be given by

$$e_1 \;=\; \frac{\sqrt{v_1}}{2T}\,. \tag{4.33}$$

eq. (4.33) simplifies the equations of motion of v_1, v_2, s and T and enables one to solve them,

$$v_1 = (3 + h\,x^2)\,\frac{\alpha'}{8}\,, \tag{4.34}$$

$$\frac{v_2}{v_1} = \frac{4(1 + h\,x^2)}{-\,h\,x^4 + (3h + 4b - 5)x^2 + 15}\,, \tag{4.35}$$

$$s = \sqrt{\frac{x\,N\,W}{v_1}}\,\frac{h\,x^4\,+\,1}{3\,+\,(b-1)\,x^2}\,\frac{v_1}{v_2} \tag{4.36}$$

$$T = \sqrt{\frac{N}{W\,x}} \tag{4.37}$$

$$e_2 = \frac{1}{2}\sqrt{v_1}\,x\,T\,, \tag{4.38}$$

where x is a root of

$$(-4\,d\,-\,6\,b\,-\,h\,+\,\frac{5}{2})\,x^4\,-\,6\,(1\,-\,b)\,x^2\,+\,9 = 0\,, \tag{4.39}$$

Note that we used x as a different parametrisation of b, d, h to express the near horizon configuration in a more convenient way. Eq's (4.33)-(4.38) identify the near horizon configuration. We use the entropy formula of entropy formalism [21, 28, 29] to calculate Wald's entropy associated to the local horizon. The entropy formalism expresses Wald's entropy, S_{BH}, by

$$S_{BH} = 2\pi(\frac{\partial f}{\partial e_1}e_1 + \frac{\partial f}{\partial e_2}e_2 - f), \tag{4.40}$$

which is evaluated on the horizon. We can use (4.11)-(4.15) to write

$$S_{BH} = 2\,\pi(\frac{N}{2}\,e_1 + \frac{W}{2}\,e_2)\,=\,\pi\,\sqrt{N\,W\,x\,v_1}\,=\,\pi\sqrt{\frac{NW\alpha'}{8}}\,\sqrt{x(3 + hx^2)} \tag{4.41}$$

where we used the local horizon parameters (4.33), (4.37) and (4.38). We see that the entropy depends on h and x which are two field redefinition ambiguity parameters satisfying (4.39) . This dependence is the result of applying Wald's entropy formula on a exact solution of the truncated equations of motion. The ambiguity parameters would be preferred in such a way that the statistical and thermodynamical entropies are in agreement. To further elaborate the scheme dependence let us require (4.41) to be the same as the statistical entropy. This equality happens in the schemes where

$$x\,v_1 = \alpha' \tag{4.42}$$

and we choose these schemes. There exist a set of ranges for the parameters of the field redefinition ambiguity where v_1, v_2, T, s are all positive. It is straightforward to identify these ranges. Let us focus on the subset of the parameters where identity is a root of (4.39) or equivalently $h = -4d + \frac{11}{2}$. In this subset T-duality in the y direction (2.8) remains trivial in the sense that interchanging N and W describes T-duality both at asymptotic

JHEP10(2006)070

infinity and near the horizon.[13] Then using (4.42) for $x = 1$ fixes d to $d = \frac{1}{8}$ for which the near horizon configuration is simplified to

$$
\begin{aligned}
v_1 &= 16, && (4.43)\\
\frac{v_2}{v_1} &= \frac{6}{5}, && (4.44)\\
T &= \sqrt{\frac{N}{W}}, && (4.45)\\
e_1 &= 2\sqrt{\frac{W}{N}}, && (4.46)\\
e_2 &= 2\sqrt{\frac{N}{W}}, && (4.47)\\
s &= \frac{5}{8}\sqrt{N\,W}, && (4.48)
\end{aligned}
$$

and we have chosen $b = 0$ and used the unit of $\alpha' = 16$. We see that $(\frac{v_1}{\alpha'}, \frac{v_2}{\alpha'}) \sim (1,1)$, and the stretched horizon is not larger than α'. We can choose other values for the field redefinition ambiguity parameters to make the local horizon arbitrarily large. For example we can choose $x = \frac{1}{2}, b = 0, h = 52, d = \frac{141}{8}$ to get

$$
\begin{aligned}
v_1 &= 2\,\alpha', && (4.49)\\
v_2 &= \frac{224}{99}\,\alpha', && (4.50)\\
T &= \sqrt{2\,\frac{N}{W}}, && (4.51)\\
e_1 &= \frac{1}{2}\sqrt{\frac{\alpha' W}{N}}, && (4.52)\\
e_2 &= \frac{1}{2}\sqrt{\frac{\alpha' N}{W}}, && (4.53)\\
s &= \frac{9}{4}\sqrt{\frac{N\,W}{\alpha'}}, && (4.54)
\end{aligned}
$$

for which one can argue that the higher order α' corrections are suppressed outside the horizon and the higher order α' corrections only provide perturbations around the "black hole". We expect that there exist schemes[14] in which Wald's entropy for a black hole of a general dimension is in agreement with the statistical entropy and $(\frac{v_1}{\alpha'}, \frac{v_2}{\alpha'}) > (1,1)$, therefore, the higher order α' corrections could be ignored outside the stretched horizon within these schemes. However we notice that the values of the field redefinition parameters are not small in these schemes. For the case of the WZW models where the exact conformal theory is known the values of the field redefinition ambiguity in which the background fields

[13] In general requiring T-duality to commute with α' corrections identifies corrections to T-duality [75, 76]. The explicit form of the α' corrections to T-duality on backgrounds composed of a diagonal metric and the dilaton is presented in [77, 78].

[14] [79] has found the local horizon configuration parameters in a general dimension for a general Lovelock gravity.

JHEP10(2006)070

retain their forms are at order one [80]. Thus it is unlikely that the large values for the field redefinition ambiguity parameters are going to be chosen by the underlying conformal field theory.

Note that there exist field redefinition ambiguities which vanish near the horizon and infinity. The class of the field redefinitions that leave the equations of the metric and NS two-form as second order differential equations is

$$T_{ij} = c_1 \boldsymbol{\nabla}_i \boldsymbol{\nabla}_j \boldsymbol{\phi} + c_2 \boldsymbol{g}_{ij} \Box \boldsymbol{\phi} + c_3 \boldsymbol{\nabla}_i \boldsymbol{\phi} \boldsymbol{\nabla}_j \boldsymbol{\phi} + c_4 \boldsymbol{g}_{ij} |\boldsymbol{\nabla}\boldsymbol{\phi}|^2 \tag{4.55}$$

$$X = c_5 \Box \boldsymbol{\phi} + c_6 |\boldsymbol{\nabla}\boldsymbol{\phi}|^2 \tag{4.56}$$

where $c_1, c_2, \ldots, c_6$ are arbitrary real numbers. ref. [37, 81] have looked for a numerical interpolating solution in one single set of the ambiguity parameters. One should study if there exists any set of values for $b, d, e, f, c_1, \ldots, c_6$ for which a smooth solution interpolates from the near horizon geometry to infinity. This question needs further investigation, however due to the large numbers of the free parameters it is tempting to argue that the interpolating solution exists in general.

The inclusion of the gravitational Chern-Simons terms does not change the fact that the entropy associated to the induced horizon is scheme dependent. Thus we expect that the inclusion of the gravitational Chern-Simons terms only results to a change in the values of the ambiguity parameters that are preferred by the equality of the statistical and thermodynamical entropies. It is intersting to apply techniques which were introduced in ref. [24, 85] in order to also include the gravitational Chern-Simons term.

5. Conclusions

We have studied the linear α' corrections and the field redefinition ambiguities in the critical Heterotic string theory for the backgrounds representing a fundamental string wrapped around a cycle.

We have required the α' corrections to the Einstein tensor to be covariantly divergence free. This requirement has enabled us to rewrite the square of the Riemann tensor as the Gauss-Bonnet Lagrangian keeping some of the field redefinition ambiguity parameters untouched. One may ask if this requirement, similar to the ghost-freedom criterion [82], could be applied to all orders in α'. This question needs further investigation. It would be intersting to find a criterion which both fixes the remaining ambiguity parameters and gives rise to a stretched horizon for half-BPS singular backgrounds representing a wrapped fundamental string. It would be intersting to study if applying the MM-criterion [60, 66] in the presence of the gravitational Chern-Simons corrections results to the stretched horizon.

We have applied a toroidal compactification to construct all the linear α' corrections to a wrapped fundamental string. We have evaluated all the linear α' corrections however excluding the gravitational Chern-Simons term to the action on the horizon. We employed the entropy formalism to find the horizon configuration parameters and to compute the entropy. We have found that Wald's entropy for the induced horizon depends on the field redefinition ambiguity parameters. This dependence is due to identifying the induced horizon as the exact solution of the truncated equations of motion.

We have shown that there exist schemes in which the inclusions of all the linear α' corrections but the gravitational Chern-Simons term gives a horizon with geometry $AdS_2 \times S^D$ for which the statistical and thermodynamical entropies are in agreement. Within these schemes the size of the horizon is scheme dependent. It is intersting to apply techniques which are introduced in ref. [24, 85] to also include the gravitational Chern-Simons term.

This means that there exist schemes in which the α' stretched horizon is small and also there exist schemes where the α' stretched horizon does not exist at all. We do not know which scheme would be preferred or chosen by the underlying conformal field theory since it is not known what a conformal field theory (and if a unique one) represents a wrapped fundamental string.

Although we have proved the existence of the schemes in which the α' stretched horizon could be larger than the string length and for which the statistical entropy is in agreement with Wald entropy, still we find disturbing that the the thermodynamical entropy is scheme-dependent. This dependence beside the existence of a scheme with no α' corrections to a fundamental string [69, 70] may be counted on as indications to express the thermodynamical properties in term of something else, in place of the event horizon, which null-singular geometries possess instead of requiring the subleading corrections to covert the null singular backgrounds to black holes with a regular event horizon. We would like to point out that Mathur and Lunin description for the entropy [46] may be employed to generate a thermodynamical entropy for a wrapped fundamental string without first requiring the α' corrections to produce an event horizon covering the singularity.

Acknowledgments

I would like to thank Loriano Bonora and Martin O'Loughlin for useful discussions and suggestions throughout the work, and careful reading of the draft. I thank Ashoke Sen and Marco Serone for their comments when I was presenting an early version of this work and Predrag Prester for correspondence. I thank Justin David for discussions.

References

[1] A.W. Peet, *Entropy and supersymmetry of D-dimensional extremal electric black holes versus string states*, *Nucl. Phys.* **B 456** (1995) 732 [hep-th/9506200].

[2] A. Dabholkar, *Exact counting of black hole microstates*, *Phys. Rev. Lett.* **94** (2005) 241301 [hep-th/0409148].

[3] A. Dabholkar, R. Kallosh and A. Maloney, *A stringy cloak for a classical singularity*, *JHEP* **12** (2004) 059 [hep-th/0410076].

[4] H. Ooguri, A. Strominger and C. Vafa, *Black hole attractors and the topological string*, *Phys. Rev.* **D 70** (2004) 106007 [hep-th/0405146].

[5] B. de Wit, $N = 2$ *electric-magnetic duality in a chiral background*, *Nucl. Phys.* **49** *(Proc. Suppl.)* (1996) 191 [hep-th/9602060].

[6] B. de Wit, $N = 2$ *symplectic reparametrizations in a chiral background*, *Fortschr. Phys.* **44** (1996) 529 [hep-th/9603191].

[7] K. Behrndt et al., *Higher-order black-hole solutions in $N = 2$ supergravity and Calabi-Yau string backgrounds*, *Phys. Lett.* **B 429** (1998) 289 [hep-th/9801081].

[8] G. Lopes Cardoso, B. de Wit and T. Mohaupt, *Corrections to macroscopic supersymmetric black-hole entropy*, *Phys. Lett.* **B 451** (1999) 309 [hep-th/9812082].

[9] G. Lopes Cardoso, B. de Wit and T. Mohaupt, *Deviations from the area law for supersymmetric black holes*, *Fortschr. Phys.* **48** (2000) 49 [hep-th/9904005].

[10] G. Lopes Cardoso, B. de Wit and T. Mohaupt, *Macroscopic entropy formulae and non-holomorphic corrections for supersymmetric black holes*, *Nucl. Phys.* **B 567** (2000) 87 [hep-th/9906094].

[11] G. Lopes Cardoso, B. de Wit and T. Mohaupt, *Area law corrections from state counting and supergravity*, *Class. and Quant. Grav.* **17** (2000) 1007 [hep-th/9910179].

[12] T. Mohaupt, *Black hole entropy, special geometry and strings*, *Fortschr. Phys.* **49** (2001) 3 [hep-th/0007195].

[13] G. Lopes Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, *Stationary BPS solutions in $N = 2$ supergravity with R^2 interactions*, *JHEP* **12** (2000) 019 [hep-th/0009234].

[14] G. Lopes Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, *Examples of stationary BPS solutions in $N = 2$ supergravity theories with R^2-interactions*, *Fortschr. Phys.* **49** (2001) 557 [hep-th/0012232].

[15] R.M. Wald, *Black hole entropy in the Noether charge*, *Phys. Rev.* **D 48** (1993) 3427 [gr-qc/9307038].

[16] V. Iyer and R.M. Wald, *Some properties of Noether charge and a proposal for dynamical black hole entropy*, *Phys. Rev.* **D 50** (1994) 846 [gr-qc/9403028].

[17] T. Jacobson, G. Kang and R.C. Myers, *Black hole entropy in higher curvature gravity*, gr-qc/9502009.

[18] D. Bak, S. Kim and S.-J. Rey, *Exactly soluble BPS black holes in higher curvature $N = 2$ supergravity*, hep-th/0501014.

[19] A. Sen, *Black holes, elementary strings and holomorphic anomaly*, *JHEP* **07** (2005) 063 [hep-th/0502126].

[20] A. Sen, *Black holes and the spectrum of half-BPS states in $N = 4$ supersymmetric string theory*, *Adv. Theor. Math. Phys.* **9** (2005) 527 [hep-th/0504005].

[21] A. Sen, *Black hole entropy function and the attractor mechanism in higher derivative gravity*, *JHEP* **09** (2005) 038 [hep-th/0506177].

[22] G.L. Cardoso, D. Lüst and J. Perz, *Entropy maximization in the presence of higher-curvature interactions*, *JHEP* **05** (2006) 028 [hep-th/0603211].

[23] A. Dabholkar, F. Denef, G.W. Moore and B. Pioline, *Precision counting of small black holes*, *JHEP* **10** (2005) 096 [hep-th/0507014].

[24] B. Sahoo and A. Sen, *Higher derivative corrections to non-supersymmetric extremal black holes in $N = 2$ supergravity*, *JHEP* **09** (2006) 029 [hep-th/0603149].

[25] M. Alishahiha and H. Ebrahim, *Non-supersymmetric attractors and entropy function*, *JHEP* **03** (2006) 003 [hep-th/0601016].

JHEP10(2006)070

[26] B. Chandrasekhar, *Born-Infeld corrections to the entropy function of heterotic black holes*, hep-th/0604028.

[27] A. Ghodsi, *R^4 corrections to d1d5p black hole entropy from entropy function formalism*, hep-th/0604106.

[28] A. Sen, *How does a fundamental string stretch its horizon?*, *JHEP* **05** (2005) 059 [hep-th/0411255].

[29] A. Sen, *Entropy function for heterotic black holes*, *JHEP* **03** (2006) 008 [hep-th/0508042].

[30] C.M. Hull and P.K. Townsend, *The two loop beta function for sigma models with torsion*, *Phys. Lett.* **B 191** (1987) 115.

[31] R.R. Metsaev and A.A. Tseytlin, *Order α' (two loop) equivalence of the string equations of motion and the sigma model Weyl invariance conditions: dependence on the dilaton and the antisymmetric tensor*, *Nucl. Phys.* **B 293** (1987) 385.

[32] T. Jacobson, G. Kang and R.C. Myers, *On black hole entropy*, *Phys. Rev.* **D 49** (1994) 6587 [gr-qc/9312023].

[33] C. Johnson, *D-branes*, Cambridge university press, 2003.

[34] J. Maharana and J.H. Schwarz, *Noncompact symmetries in string theory*, *Nucl. Phys.* **B 390** (1993) 3 [hep-th/9207016].

[35] S.F. Hassan and A. Sen, *Twisting classical solutions in heterotic string theory*, *Nucl. Phys.* **B 375** (1992) 103 [hep-th/9109038].

[36] A. Sen, *Black hole solutions in heterotic string theory on a torus*, *Nucl. Phys.* **B 440** (1995) 421 [hep-th/9411187].

[37] A. Sen, *Stretching the horizon of a higher dimensional small black hole*, *JHEP* **07** (2005) 073 [hep-th/0505122].

[38] D. Kutasov, *Accelerating branes and the string/black hole transition*, hep-th/0509170.

[39] V. Kazakov, I.K. Kostov and D. Kutasov, *A matrix model for the two-dimensional black hole*, *Nucl. Phys.* **B 622** (2002) 141 [hep-th/0101011].

[40] A. Dabholkar and J.A. Harvey, *Nonrenormalization of the superstring tension*, *Phys. Rev. Lett.* **63** (1989) 478.

[41] A. Strominger and C. Vafa, *Microscopic origin of the Bekenstein-Hawking entropy*, *Phys. Lett.* **B 379** (1996) 99 [hep-th/9601029].

[42] C. Vafa, *Black holes and Calabi-Yau threefolds*, *Adv. Theor. Math. Phys.* **2** (1998) 207 [hep-th/9711067].

[43] J.M. Maldacena, A. Strominger and E. Witten, *Black hole entropy in M-theory*, *JHEP* **12** (1997) 002 [hep-th/9711053].

[44] M. W. Robert, *Quantum field theory in curved spacetime and black hole thermodynamics*, The University of Chicago Press, 1994, p. 165.

[45] G.W. Gibbons and S.W. Hawking, *Action integrals and partition functions in quantum gravity*, *Phys. Rev.* **D 15** (1977) 2752.

[46] O. Lunin and S.D. Mathur, *Statistical interpretation of Bekenstein entropy for systems with a stretched horizon*, *Phys. Rev. Lett.* **88** (2002) 211303 [hep-th/0202072].

JHEP10(2006)070

[47] V.S. Rychkov, *D1-D5 black hole microstate counting from supergravity*, *JHEP* **01** (2006) 063 [hep-th/0512053].

[48] C.G. Callan Jr., E.J. Martinec, M.J. Perry and D. Friedan, *Strings in background fields*, *Nucl. Phys.* **B 262** (1985) 593.

[49] Y.M. Cho and I.P. Neupane, *Anti-de Sitter black holes, thermal phase transition and holography in higher curvature gravity*, *Phys. Rev.* **D 66** (2002) 024044 [hep-th/0202140].

[50] D.J. Gross and J.H. Sloan, *The quartic effective action for the heterotic string*, *Nucl. Phys.* **B 291** (1987) 41.

[51] C.G. Callan Jr., I.R. Klebanov and M.J. Perry, *String theory effective actions*, *Nucl. Phys.* **B 278** (1986) 78.

[52] C. Lovelace, *Stability of string vacua, 1. A new picture of the renormalization group*, *Nucl. Phys.* **B 273** (1986) 413.

[53] A. Sen, *The heterotic string in arbitrary background field*, *Phys. Rev.* **D 32** (1985) 2102.

[54] A. Sen, *Equations of motion for the heterotic string theory from the conformal invariance of the sigma model*, *Phys. Rev. Lett.* **55** (1985) 1846.

[55] C.M. Hull and P.K. Townsend, *Finiteness and conformal invariance in nonlinear sigma models*, *Nucl. Phys.* **B 274** (1986) 349.

[56] Y. Okawa and B. Zwiebach, *Heterotic string field theory*, *JHEP* **07** (2004) 042 [hep-th/0406212].

[57] H. Yang and B. Zwiebach, *A closed string tachyon vacuum?*, *JHEP* **09** (2005) 054 [hep-th/0506077].

[58] I. Jack, D.R.T. Jones and D.A. Ross, *On the relationship between string low-energy effective actions and $O(\alpha'^3)$ sigma model beta functions*, *Nucl. Phys.* **B 307** (1988) 130.

[59] R. Brustein, D. Nemeschansky and S. Yankielowicz, *Beta functions and S matrix in string theory*, *Nucl. Phys.* **B 301** (1988) 224.

[60] I. Jack and D. Jones, *σ-model β-functions and ghost free string effective actions*, *Nucl.Phys.* **B303** (1986).

[61] M. Blau, *Lecture notes on general relativity.*

[62] D. Lovelock, *The Einstein tensor and its generalization*, *Jour.Math.Phys* **12** (1971).

[63] B. Zwiebach, *Curvature squared terms and string theories*, *Phys. Lett.* **B 156** (1985) 315.

[64] J.T. Wheeler, *Symmetric solutions to the Gauss-Bonnet extended Einstein equations*, *Nucl. Phys.* **B 268** (1986) 737.

[65] B. Zumino, *Gravity theories in more than four-dimensions*, *Phys. Rept.* **137** (1986) 109.

[66] N. Mavromates and J. Miramontes, *Effective actions from the conformal invariance conditions of Bosonic σ models with graviton and dilaton background*, *Phys.Let.* **B201** (1988) 473

[67] M.N.R. Wohlfarth, *Gravity a la Born-Infeld*, *Class. and Quant. Grav.* **21** (2004) 1927 [hep-th/0310067].

[68] D. Grumiller, *An action for the exact string black hole*, *JHEP* **05** (2005) 028 [hep-th/0501208].

[69] G.T. Horowitz and A.A. Tseytlin, *A new class of exact solutions in string theory*, *Phys. Rev.* **D 51** (1995) 2896 [hep-th/9409021].

[70] M. Cvetič and A.A. Tseytlin, *General class of BPS saturated dyonic black holes as exact superstring solutions*, *Phys. Lett.* **B 366** (1996) 95 [hep-th/9510097].

[71] G. Exirifard, *The world-sheet corrections to dyons in the heterotic theory*, hep-th/0607094.

[72] F. Mueller-Hoissen, *Nonminimal coupling from dimensional reduction of the Gauss-Bonnet action*, *Phys. Lett.* **B 201** (1988) 325.

[73] S. Ferrara, R. Kallosh and A. Strominger, $N = 2$ *extremal black holes*, *Phys. Rev.* **D 52** (1995) 5412 [hep-th/9508072].

[74] A. Strominger, *Macroscopic entropy of* $N = 2$ *extremal black holes*, *Phys. Lett.* **B 383** (1996) 39 [hep-th/9602111].

[75] P.E. Haagensen and K. Olsen, *T-duality and two-loop renormalization flows*, *Nucl. Phys.* **B 504** (1997) 326 [hep-th/9704157].

[76] P.E. Haagensen, K. Olsen and R. Schiappa, *Two-loop beta functions without feynman diagrams*, *Phys. Rev. Lett.* **79** (1997) 3573 [hep-th/9705105].

[77] G. Exirifard and M. O'Loughlin, *Two and three loop* α' *corrections to T-duality: kasner and schwarzschild*, *JHEP* **12** (2004) 023 [hep-th/0408200].

[78] G. Exirifard, *Quadratic* α' *corrections to T-duality*, *JHEP* **07** (2005) 047 [hep-th/0504133].

[79] P. Prester, *Lovelock type gravity and small black holes in heterotic string theory*, *JHEP* **02** (2006) 039 [hep-th/0511306].

[80] K. Sfetsos and A.A. Tseytlin, *Antisymmetric tensor coupling and conformal invariance in sigma models corresponding to gauged WZNW theories*, *Phys. Rev.* **D 49** (1994) 2933 [hep-th/9310159].

[81] V. Hubeny, A. Maloney and M. Rangamani, *String-corrected black holes*, *JHEP* **05** (2005) 035 [hep-th/0411272].

[82] I. Jack, D. Jones, and A. Lawrence, *Ghost freedom and string theory*, *Phys. Lett.* **B203** (1988) 378.

[83] P. Kraus and F. Larsen, *Microscopic black hole entropy in theories with higher derivatives*, *JHEP* **09** (2005) 034 [hep-th/0506176].

[84] P. Kraus and F. Larsen, *Holographic gravitational anomalies*, *JHEP* **01** (2006) 022 [hep-th/0508218].

[85] B. Sahoo and A. Sen, α' *corrections to extremal dyonic black holes in heterotic string theory*, hep-th/0608182.

JHEP10(2006)070